

Random Matrix Theory: a quick introduction

Carlos A. González



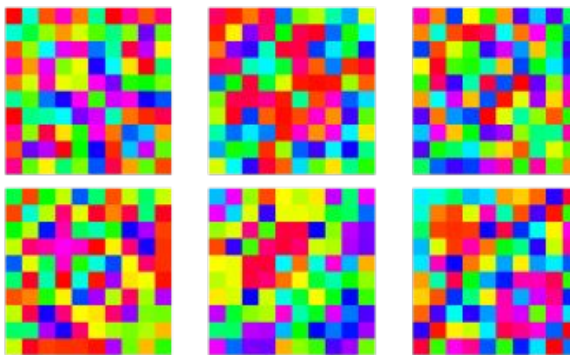
Quantum

Materials and Devices

Journal Club

What is RMT?

Expectation →



← Reality

The birth of RMT

1928: John Wishart



Wishart drowned aged 57 in July 1956, having suffered a stroke while swimming in the sea at Revolcadero Beach, [Acapulco](#) (Mexico)

$$X = \begin{array}{c} \text{phys.} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \text{math} \\ \left| \begin{array}{cc} X_{11} & X_{12} \\ X_{21} & X_{22} \\ X_{31} & X_{32} \end{array} \right| \end{array} \quad \begin{array}{l} \text{in general} \\ (M \times N) \end{array}$$

$$X^t = \begin{array}{c} \left| \begin{array}{ccc} X_{11} & X_{21} & X_{31} \\ X_{12} & X_{22} & X_{32} \end{array} \right| \end{array} \quad \begin{array}{l} \text{in general} \\ (N \times M) \end{array}$$

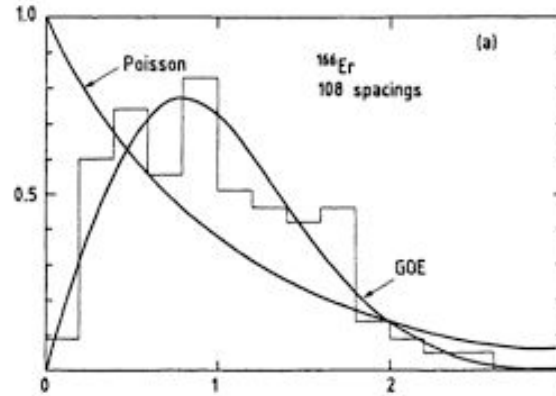
$$W = X^t X = \begin{array}{c} \left| \begin{array}{cc} X_{11}^2 + X_{21}^2 + X_{31}^2 & X_{11}X_{12} + X_{21}X_{22} + X_{31}X_{32} \\ X_{12}X_{11} + X_{22}X_{21} + X_{32}X_{31} & X_{12}^2 + X_{22}^2 + X_{32}^2 \end{array} \right| \end{array}$$

→ (N × N) COVARIANCE MATRIX (unnormalized)

Null model → random data: $X \rightarrow$ random $(M \times N)$ matrix
→ $W = X^t X \rightarrow$ random $N \times N$ matrix (Wishart, 1928)

RMT in Physics

1950: Eugene Wigner

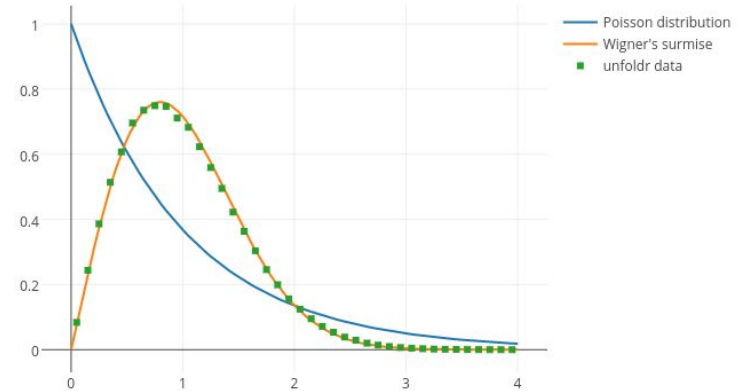


DISCUSSION

W. HAVENS: Where does one find out about Wishart distribution?

E. WIGNER: A Wishart distribution is given in S. S. Wilks book about statistics and I found it by accident.

Nearest-neighbor level spacing distribution

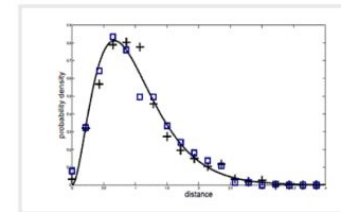


The Curious Link Between Parked Cars and Perched Birds

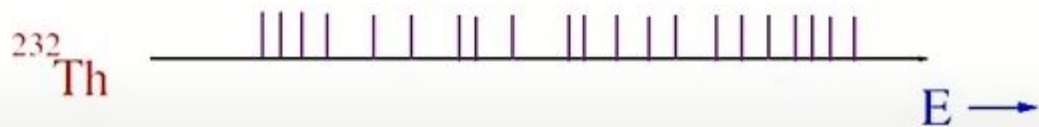
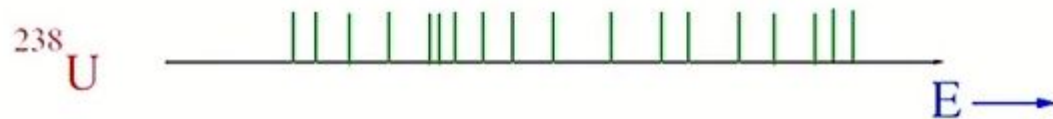
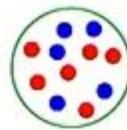
Could an uncanny resemblance between the statistics of parked cars and perched birds help us understand the relationship between mathematics and physics?

KFC 07/15/2009

5 COMMENTS



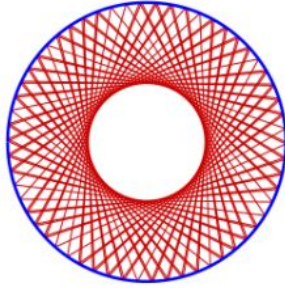
spectra of heavy nuclei



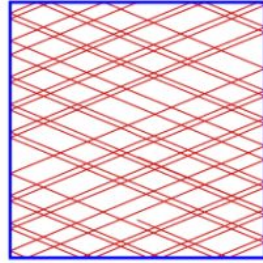
WIGNER ('50) : replace complex H by random matrix

Classical chaos

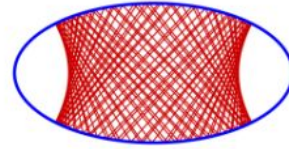
(a)



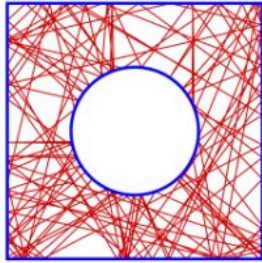
(b)



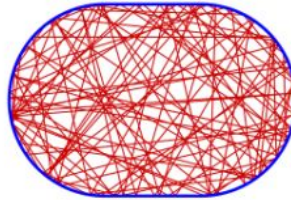
(c)



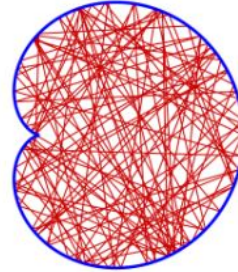
(d)



(e)

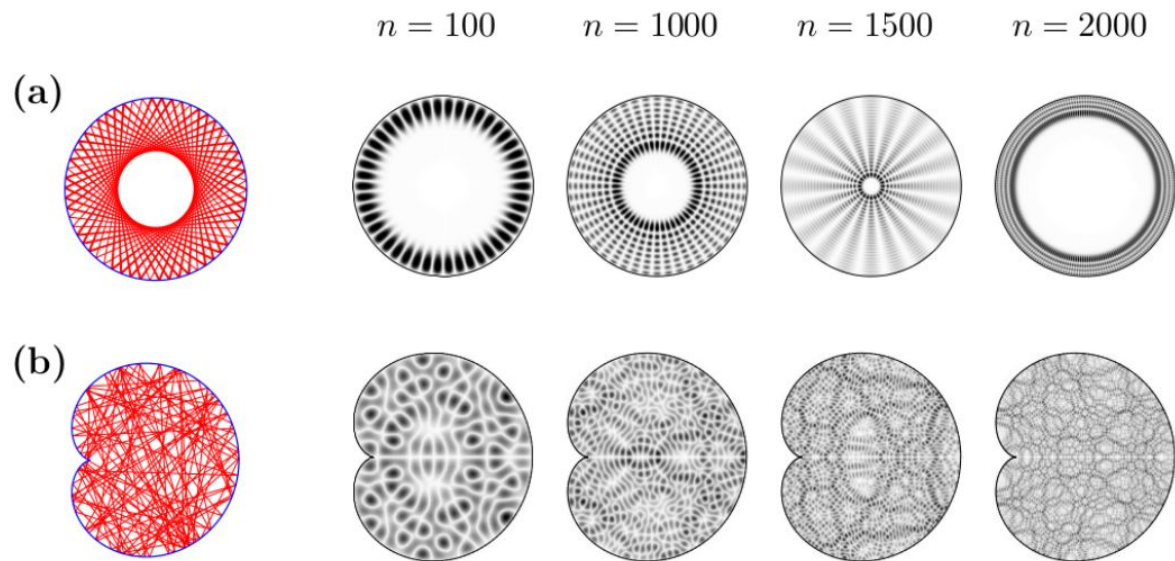


(f)



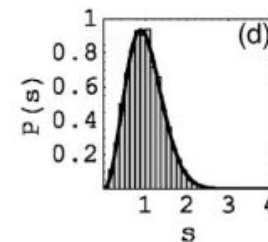
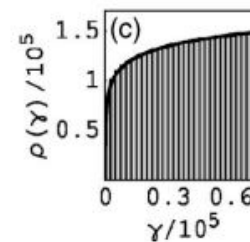
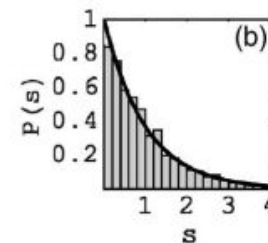
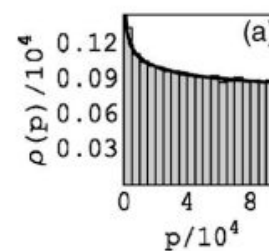
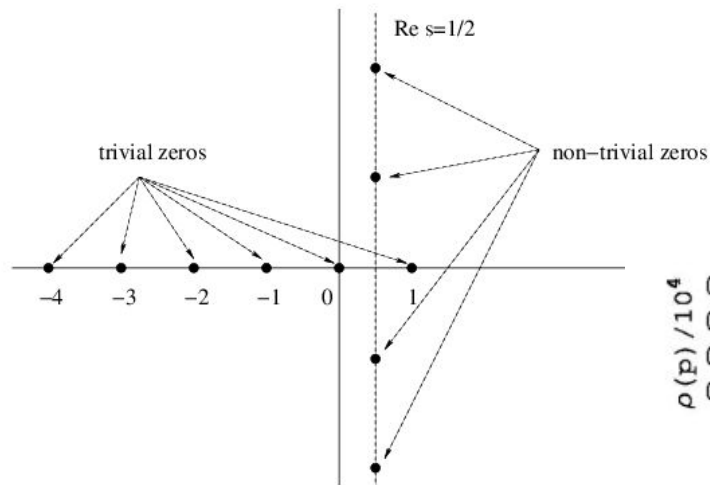
Quantum chaos

Michael Berry



RMT and number theory

B. Riemann



- World record calculation of non-trivial zeros.
- Erdos number = 1

New directions in Mathematics

Terry Tao



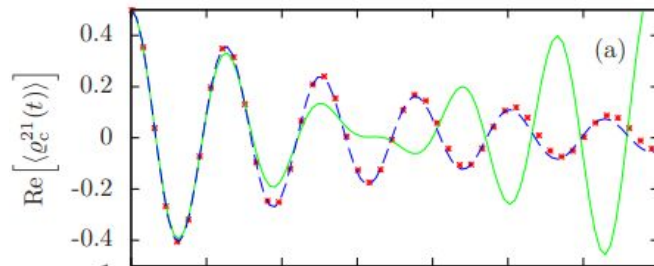
- PhD at the age of 21
- full professor of UCLA at the age of 24.
- fields medal in 2006
- Erdos number = 2



Recent applications:

qubit + generic (chaotic) bath (T. Gorin et al. Phys.Rev. A 2019)

$$H_\lambda = \frac{\Delta}{2} \sigma_z \otimes \mathbb{1}_e + \mathbb{1}_c \otimes H_e + \lambda v_c \otimes V_e.$$



Random Matrix Methods for Machine Learning: When Theory meets Applications

Romain Couillet, Zhenyu Liao

August 20, 2021

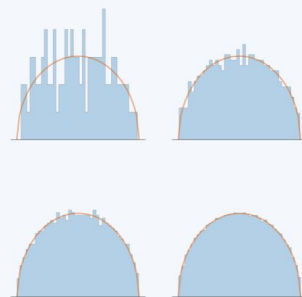
Random Matrix Theory and Machine Learning Tutorial

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ICML 2021 tutorial on

Random Matrix Theory and Machine Learning

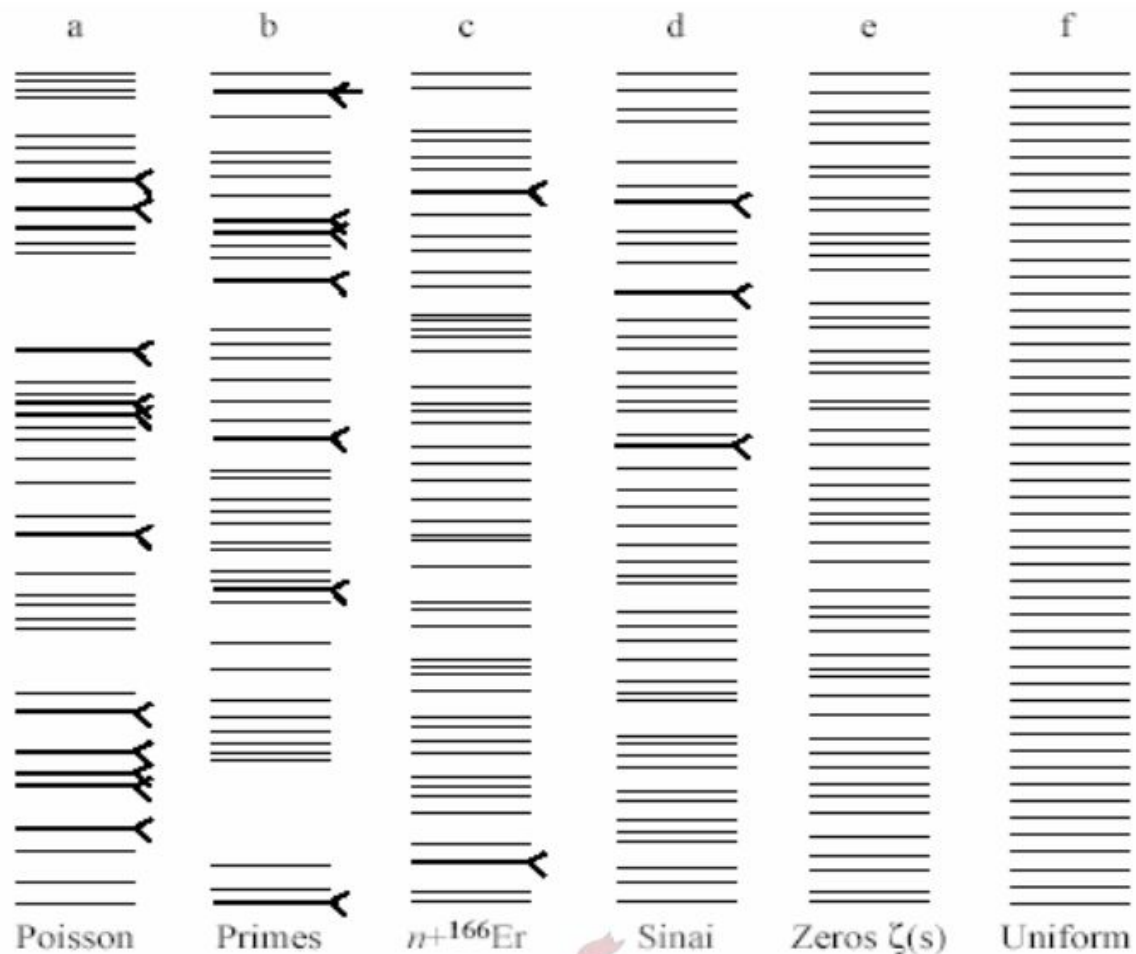
In recent years, random matrix theory (RMT) has come to the forefront of learning theory as a tool to understand some of its most important challenges. From generalization of deep learning models to a precise analysis of optimization algorithms, RMT provides analytically tractable models.

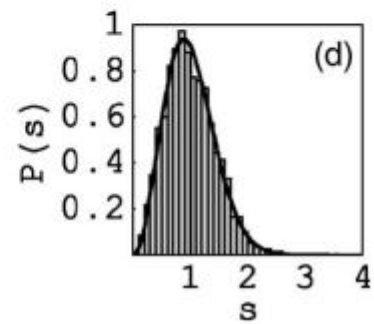
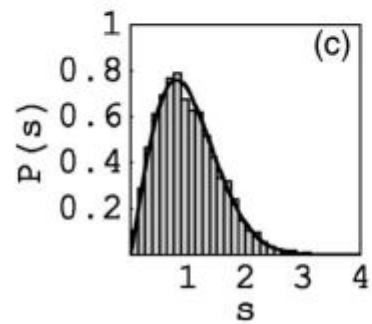
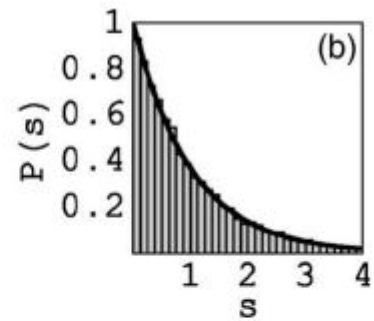
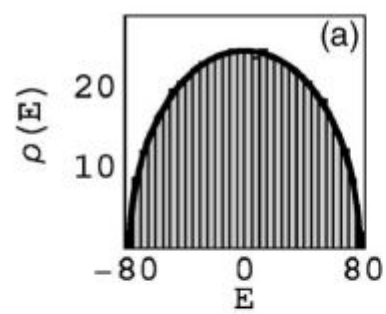


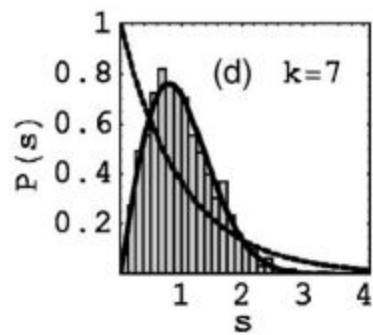
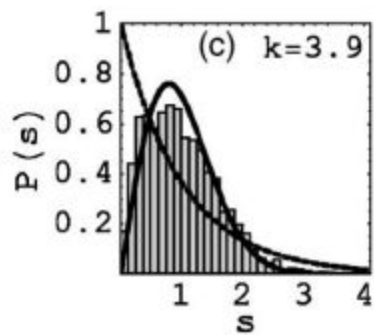
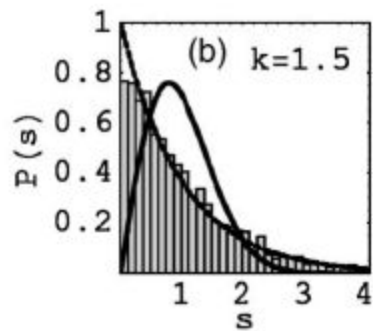
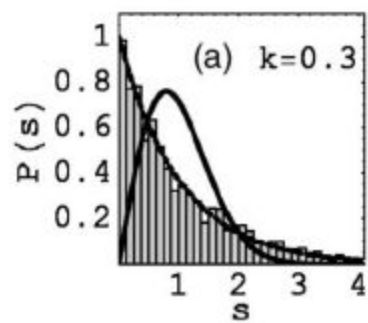
KNOW MORE >

References:

- [Hans-Jürgen Stöckmann](#), *Quantum Chaos: An Introduction*, (1999)
- Fritz Haake, *Quantum Signatures of Chaos* 2nd ed., (2001)







Spin systems

L. Santos (2012)

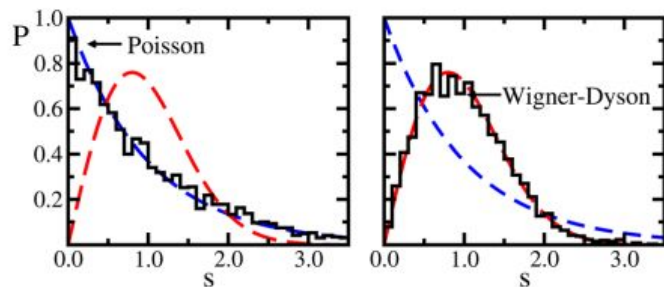


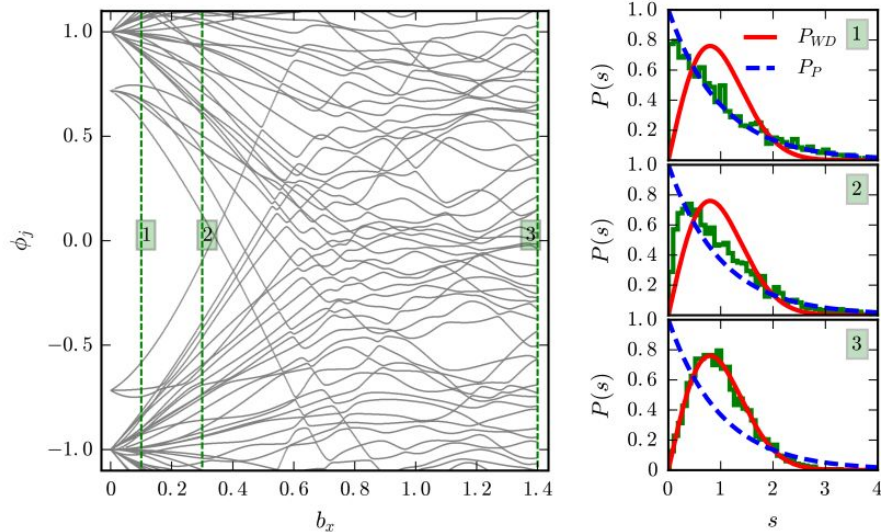
Fig. 1. (Color online) Level spacing distribution for the Hamiltonian in Eq. (1) with $L = 15$, 5 spins up, $\omega = 0$, $\epsilon_d = 0.5$, $J_{xy} = 1$, and $J_z = 0.5$ (arbitrary units); bin size = 0.1. (a) Defect on site $d = 1$; (b) defect on site $d = 7$. The dashed lines are the theoretical curves.

$$H = H_z + H_{NN},$$

$$H_z = \sum_{i=1}^L \omega_i S_i^z = \left(\sum_{i=1}^L \omega S_i^z \right) + \epsilon_d S_d^z,$$

$$H_{NN} = \sum_{i=1}^{L-1} [J_{xy} (S_i^x S_{i+1}^x + S_i^y S_{i+1}^y) + J_z S_i^z S_{i+1}^z].$$

C. Pineda, T. Prosen (2007)



$$H_{\text{KI}}(t) = \sum_{j>k}^{N_0} J_{j,k} \sigma_z^j \sigma_z^k + K(t) \sum_j^N \vec{b}_j \cdot \vec{\sigma}^j,$$